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On a Nonlinear Transaction-Cost Model for Stock Prices in an Illiquid Market Driven by a Relaxed Black-Scholes Model Assumptions

S. O. Edeki *[1], O.O. Ugbebor[1,2], and E.A. Owoloko[1]

[1]*Department of Mathematics, Covenant University, Nigeria*
[2]*Department of Mathematics, University of Ibadan, Nigeria*

E-mail: soedeki@yahoo.com
**Corresponding author*

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ABSTRACT

In an illiquid market, assets cannot be easily sold or exchanged for cash without a loss of value (even if it is minimal), this may be due to uncertainty such as transaction cost, lack of interested buyers and so on. This paper considers a nonlinear transaction-cost model for stock prices in an illiquid market. This nonlinear model surfaced when the constant volatility assumption of the famous linear Black-Scholes option valuation and pricing model is relaxed via the inclusion of transaction cost. We obtain approximate solutions to this nonlinear model using the projected differential transform technique or method (PDTM) as a semi-analytical method. The results are very interesting, agree with the associated exact solutions of Esekon (2013) and that of Gonzalez-Gaxiola et al. (2015).

Keywords: Nonlinear Black-Scholes model, illiquid market, option pricing, PDTM.

1. Introduction

In a professional setting, the term 'liquidity' describes the level to which an underlying asset can be quickly exercised- sold or bought in the market with the asset's price not affected. That is to say, that liquidity of an asset describes the flexibility and ease of the asset in terms of quick sales, with less regard to the asset's price reduction (Acharya and Pedersen (2005);Amihud and Mendelson (1986)). Examples of liquid assets include money or cash as it can be sold for items such as goods and services (immediately) without (or with minimal) loss of value. A liquid market is basically characterized by ever ready and willing investors. On the other hand (Keynes (1971)), in an illiquid market, assets cannot be easily sold or exchanged cash-wise without a noticeable reduction in price due to uncertainty such as transaction cost, lack of interested buyers, among others. In the period of market chaos when the ratio of buyers to sellers is relative not balanced, illiquid type of assets attract higher risks than liquid types. Stock option is an example of an illiquid asset.

In the study of modern finance and pricing theory, the standard Black-Scholes model appear very useful (Black and Scholes (1973)). Though, most of the assumptions under which this classical arbitrage pricing theory is formulated seem not realistic in practice. These include: the asset price or the underlying asset following a Geometric Brownian Motion (GBM), the drift parameter and the volatility rate are assumed constants, lack of arbitrage opportunities (no risk-free profit), frictionless and competitive markets (Gonzalez-Gaxiola et al. (2015) and Owoloko and Okeke (2014)). In a competitive market, there are no transaction costs (say taxes), and trade restrictions are not honoured (say short sale constraints) (Cetin et al. (2004)), while in a competitive market, a trader is free to purchase or sell any amount of a security without altering the prices.

Based on the above assumptions, the price of the stock S, at time $t\,(0 < t < T)$ follows the stochastic differential equation (SDE):

$$dS = S\left(\mu dt + \sigma dW_t\right) \tag{1}$$

where μ, σ and W_t are mean rate of return of S, the volatility, and a standard Brownian motion respectively.

For an option value $u = u(s,t)$, we have:

$$\frac{\partial u}{\partial t} + rS\frac{\partial u}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 u}{\partial S^2} - ru = 0 \tag{2}$$

with $u\left(0,t\right) = 0, u\left(s,t\right) \to 0$ as $S \longrightarrow \infty$ $u\left(s,T\right) = max\left(S-E,0\right), E$ is a constant.

A good number of models with respect to volatility have been proposed in literature for option pricing. The simplest of them assumes constant volatility. However, it is obvious that constant volatility cannot fully explain observed market prices for options valuation unless when modified (Edeki et al. (2016a); Barles and Soner (1998); Edeki et al. (2016b); Boyle and Vorst (1992)). Equation 1 is a linear partial differential equation (the classical Black-Scholes model). Many researchers have attempted solving equation 2 for analytical or approximate solutions using direct, analytical or semi-analytical methods (Ankudinova and Ehrhardt (2008); Allahviranloo and Behzadi (2013); Jdar et al. (2005); Rodrigo and Mamon (2006); Bohner and Zheng (2009); Company et al. (2008); Cen and Le (2011);Edeki et al. (2015)). On relaxing the frictionless and the competitive markets' assumptions, the notion of liquidity is therefore introduced, giving rise to a nonlinear version of the Black-Scholes model (as a result of transaction cost involvement). Bakstein and Howison (2003) referred to liquidity as the act of grouping individual trader's transaction cost in line with the effect of price slippage. It is therefore, our intention to obtain an analytical solution of the nonlinear transaction cost-model for stock prices in an illiquid market.

2. The Nonlinear Black-Scholes model (Bakstein and Howison equation)

Here, we will consider a case where both the drift μ , and the volatility σ , parameters can be expressed as functions of the following: time τ , stock price S , and the differential coefficients of the option price V . In particular, that of non-constant modified function:

$$\sigma = \hat{\sigma}\left(\tau, S, \frac{\partial V}{\partial S}, \frac{\partial^2 V}{\partial S^2}\right) \tag{3}$$

is to be considered. So, (1) becomes:

$$\frac{\partial V}{\partial \tau} + rS\frac{\partial V}{\partial S} + \frac{1}{2}S^2\hat{\sigma^2}\left(\tau, S.\frac{\partial V}{\partial S}, \frac{\partial^2 V}{\partial S^2}\right)\frac{\partial^2 V}{\partial S^2} - rV = 0 \tag{4}$$

Note: the model equation 2 can be improved using (Equation 3) from the aspect of transaction costs inclusion, large trader and illiquid markets effect. In this regard, we will follows the approach of Frey and Patie (2002) and Frey and Stremme (1997) for the effects on the price with the result:

$$\sigma = \hat{\sigma}\left(\tau, S, \frac{\partial V}{\partial S}, \frac{\partial^2 V}{\partial S^2}\right)\left(1 - \rho S\lambda\left(S\right)\frac{\partial^2 V}{\partial S^2}\right) \tag{5}$$

where σ is the traditional volatility, ρ is a constant measuring the liquidity of the market and λ is the price of risk.

Following the assumption that the price of risk is unity (a special case where $\lambda\left(S\right) = 1$, and a little algebra with the notion that:

$$1 \approx ((1 - f_*)^2\left(1 + 2f_* + O(f_*)^3\right).$$

We can therefore write 4:

$$\frac{\partial V}{\partial \tau}+rS\frac{\partial V}{\partial S}+\frac{1}{2}S^2\left[\sigma^2\left(1+2\rho S\frac{\partial^2 V}{\partial S^2}\right)\right]\frac{\partial^2 V}{\partial S^2}-rV=0 \tag{6}$$

such that $V(S,T)=h(S), S\in[0,\infty)$. For the translation $t+\tau=T$ and using $w(S,t)=V(S,\tau)$, equation 6 becomes:

$$\frac{\partial w}{\partial t}+rS\frac{\partial w}{\partial S}=\frac{1}{2}S^2\sigma^2\left(1+2\rho S\frac{\partial^2 w}{\partial S^2}\right)\frac{\partial^2 w}{\partial S^2}-rw=0, w(S,0)=h(S) \tag{7}$$

Equation7 has an exact solution (Esekon (2013)) of the form:

$$w(S,t)=w=S-\rho^{-1}\sqrt{S_0}\left(\sqrt{S}exp\left(\frac{r+\frac{\sigma^2}{4}}{2}\right)t+\frac{\sqrt{S_0}}{4}exp\left(r+\frac{\sigma^2}{4}\right)t\right) \tag{8}$$

For $\sigma, S_0, S, \rho>0)$ while $r,t\geq 0, S_0$ as an initial stock price, with

$$w(S,0)=max\left(S-\rho^{-1}\left(\sqrt{S_0S}+\frac{S_0}{4}\right),0\right) \tag{9}$$

Remark: We note here that for$\rho=0$, equation 2 is obtained. Existence and uniqueness of this nonlinear model has been established in Liu and Yong (2005).

3. The Overview of the PDT Method

Here, an outline of the modified form of the DTM known as PDTM will be presented (Jang (2010); Edeki et al. (2016c); Ravi Kanth and Aruna (2012) and Keskin et al. (2011)).

3.1 A note on some basic theorems of the PDTM

In consideration, let $u(x,t)$ be an analytic function at (x^*,t^*) defined on a domain D^* , so considering the expansion of $u(x,t)$ in Taylor series form, we give regard to some variables $S_v=t$, unlike the approach in the classical DTM where all the variables are considered. So, the PDTM of $u(x,t)$ with respect to t at t^* is defined and denoted as follows:

$$U(x,h)=\frac{1}{h!}\left[\frac{\partial^h u(x,t)}{\partial t^h}\right]_{t=t^*} \tag{10}$$

such that

$$u(x,h)=\sum_{h=0}^{\infty}U(x,h)(t-t^*)^h \tag{11}$$

where equation 11 is called an inverse projected differential transform (IPDT) of $U(x,h)$ with respect to(time parameter).

3.1.1 Some Basic Properties and theorems of the PDTM.

a: If $m(x,t)=\alpha m_a(x,t)+\beta m_b(x,t)$, then $M(x,\bar{h})=\alpha M_a(x,\bar{h})+\beta M_b(x,\bar{h})$

b: If $m(x,t)=\alpha\dfrac{\partial^n m_*(x,t)}{\partial t^n}$, then $m(x,\bar{h})=\alpha\dfrac{(\bar{h}+n)!}{\bar{h}!}M_*(x,\bar{h}+n)$

c: If $m(x,t)=\alpha\dfrac{\partial M_*(x,t)}{\partial t}$, then $M(x,\bar{h})=\dfrac{\alpha(\bar{h}+1)!M_*(x,\bar{h}+1)}{\bar{h}!}$

d: If $m(x,t)=\vartheta(x)\dfrac{\partial^n m_*(x,t)}{\partial x^n}$, then $M(x,\bar{h})=\vartheta(x)\dfrac{\partial^n M_*(x,\bar{h})}{\partial x^n}$

e: If $m(x,t)=\vartheta(x)m^2(x,t)$, then $M(x,\bar{h})=\vartheta(x)\displaystyle\sum_{r=0}^{\bar{h}}M_*(x,r)M_*(x,\bar{h}-r)$

f: If $p(x,y)=x^r y^{r^*}$, then $P(k,\bar{h})=\delta(k-r,\bar{h}-r^*)=\delta(k-r)\delta(\bar{h}-r^*)$

where

$$\delta(k-r)=\begin{cases}1, & \text{if k = r}\\ 0, & \text{if k} \neq \text{r}\end{cases} \quad \& \quad \delta(k-r^*)=\begin{cases}1, & \text{if k} = r^*\\ 0, & \text{if k} \neq r^*\end{cases}$$

Thus,

$$u(x,t)=\sum_{\bar{h}=0}^{\infty}U(x,\bar{h})t^{\bar{h}} \tag{12}$$

4. The PDTM and the Nonlinear Model

In this subsection, the PDTM approach will be applied to the model equation 7 as follows:

$$\frac{\partial w}{\partial t}=-rS\frac{\partial w}{\partial S}-\frac{1}{2}S^2\sigma^2\left(1+2\rho S\frac{\partial^2 w}{\partial S^2}\right)\frac{\partial^2 w}{\partial S^2}+rw \tag{13}$$

subject to: $\quad w(S,0)=max\left(S-\rho^{-1}\left(\sqrt{S_0 S}+\dfrac{S_0}{4}\right),0\right)$

$$\frac{\partial w}{\partial t}=-\left(rS\frac{\partial w}{\partial S}+\frac{1}{2}S^2\sigma^2\left(\frac{\partial^2 w}{\partial S^2}+2\rho S\left(\frac{\partial^2 w}{\partial S^2}\right)^2\right)-rw\right) \tag{14}$$

At projection, the transformation of equation 14 using PDTM yields:

$$(k+1)W_{k+1}(S) = -\left(rSW_k'(S) + \frac{1}{2}S^2\sigma^2 H - rW_k(S)\right), \tag{15}$$

where

$$H = \left(W_k^{''}(S) + 2\rho S \sum_{n=0}^{k} W_n^{''}(S)W_{k-n}^{''}(S)\right) \tag{16}$$

We write equation 15 for $W_{k+1} = W_{k+1}(S)$ as:

$$W_{k+1} = \frac{-1}{k+1}\left(rSW_k' + \frac{1}{2}S^2\sigma^2\left(W_k^{''} + 2\rho S\sum_{n=0}^{k} W_n^{''}W_{k-n}^{''}\right) - rW_k\right) \tag{17}$$

$$\text{subject to :}\quad W_0 = max\left(S - \rho^{-1}\left(\sqrt{S_0 S} + \frac{S_0}{4}\right), 0\right) \tag{18}$$

when $k = 0$.

$$W_1 = -\left(rSW_0' + \frac{1}{2}S^2\sigma^2(W_0^{''} + 2\rho SW_0^{''}W_0^{''}) - rW_0\right) \tag{19}$$

when $k = 1$,

$$\begin{aligned} W_2 &= \tfrac{-1}{2}\left(rSW_1' + \tfrac{1}{2}S^2\sigma^2\left(W_1^{''} + 2\rho S\textstyle\sum_{n=0}^{1} W_n^{''}W_{1-n}^{''}\right) - rW_1\right) \\ &= \tfrac{-1}{2}\left(rSW_1' + \tfrac{1}{2}S^2\sigma^2(W_1^{''} + 2\rho S(W_0^{''}W_1^{''} + W_1^{''}W_0^{''})) - rW_1\right) \end{aligned} \tag{20}$$

when $k = 2$,

$$W_3 = \frac{-1}{3}\left(rSW_2' + \frac{1}{2}S^2\sigma^2\left(W_2^{''} + 2\rho S\sum_{n=0}^{2} W_n^{''}W_{2-n}^{''}\right) - rW_2\right) \tag{21}$$

when $k = 3$,

$$W_4 = \frac{-1}{4}\left(rSW_3' + \frac{1}{2}S^2\sigma^2\left(W_3^{''} + 2\rho S\sum_{n=0}^{3} W_n^{''}W_{3-n}^{''}\right) - rW_3\right) \tag{22}$$

when $k = 4$.

$$W_5 = \frac{-1}{5}\left(rSW_4' + \frac{1}{2}S^2\sigma^2\left(W_4^{''} + 2\rho S\sum_{n=0}^{4} W_n^{''}W_{4-n}^{''}\right) - rW_4\right) \tag{23}$$

4.1 Numerical Illustration

We recall (8) and (9) as follows;

$$w(S,t) = w = S - \rho^{-1}\sqrt{S_0}\left(\sqrt{S}exp\left(\frac{r+\frac{\sigma^2}{4}}{2}\right)t + \frac{\sqrt{S_0}}{4}exp\left(r+\frac{\sigma^2}{4}\right)t\right) \tag{24}$$

$$w(S,0) = max\left(S - \rho^{-1}\left(\sqrt{S_0 S} + \frac{S_0}{4}\right), 0\right) \tag{25}$$

For numerical computation, the following cases will be considered:

Case I: For $r = 0, \rho = -0.01, \sigma = 0.4, S_0 = 4$ we thus have the exact solution and initial condition as:

$$w(S,t) = S + 200\left(\sqrt{S}exp\left(\frac{t}{50}\right) + \frac{1}{2}exp\left(\frac{t}{100}\right)\right) \tag{26}$$

$$w(S,0) = S + 200\sqrt{S} + 100 \tag{27}$$

So, applying the PDTM with the parameters in case I through (17)-(23) gives the following:

$$W(S,0) = S + 200\sqrt{S} + 100 \tag{28}$$

$$W(S,1) = -\frac{S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right) \tag{29}$$

$$\begin{aligned} W(S,2) = &-\frac{S^2}{25}\left(-\frac{8}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right) - \frac{4S}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2\right. \\ &\left. - \frac{4S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} + \frac{300}{S^4}\right)\right. \\ &- \frac{S}{50}\left(\frac{-50\left(\frac{-4}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2 - \frac{4}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} - \frac{300}{S^4}\right)\right)}{S^{\frac{3}{2}}}\right) \\ &- \frac{1}{S^{\frac{3}{2}}}\left(50\left(-\frac{8}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right) - \frac{4S}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2\right.\right. \\ &\left.\left. - \frac{4S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} + \frac{300}{S^4}\right)\right)\right)\Bigg)\Bigg) \end{aligned}$$

whence,

$$
\begin{aligned}
w(S,t) =& \sum_{h=0}^{\infty} W(S,h)t^h \\
=& W(S,0) + W(S,1)t + W(S,2)t^2 + W(S,h)t^3 + \cdots \\
=& \left(S + 200\sqrt{S} + 100\right) - \frac{S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)t \\
& + \left(-\frac{S^2}{25}\left(-\frac{8}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right) - \frac{4S}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2\right.\right. \\
& - \frac{4S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} + \frac{300}{S^4}\right) \\
& - \frac{S}{50}\left(\frac{-50\left(\frac{-4}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2 - \frac{4}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} - \frac{300}{S^4}\right)\right)}{S^{\frac{3}{2}}}\right) \\
& - \frac{1}{S^{\frac{3}{2}}}\left(50\left(-\frac{8}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right) - \frac{4S}{25}\left(\frac{75}{S^{\frac{5}{2}}} + \frac{100}{S^3}\right)^2\right.\right. \\
& \left.\left.\left.\left.\left. - \frac{4S}{25}\left(\frac{-50}{S^{\frac{3}{2}}} - \frac{50}{S^2}\right)\left(\frac{-375}{2S^{\frac{7}{2}}} + \frac{300}{S^4}\right)\right)\right)\right)\right)\right)t^2 + \cdots
\end{aligned}
$$

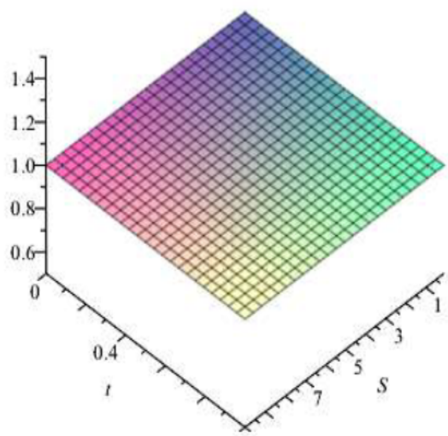

Figure 1: Approximate solution for problem case I

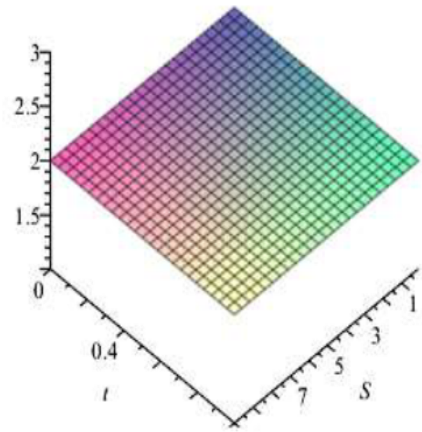

Figure 2: Exact solution for problem case I

Figures 1 and 2 above are the graphics for approximate and exact solutions for*problem case I* respectively, for $S \in [0.1, 10]$ and $t \in [0, 1]$.

Case II: For $r = 0.06, \rho = -0.01, \sigma = 0.4, S_0 = 4$, we thus have the exact solution and initial condition as:

$$w(S,t) = S + 200\left(\sqrt{S}exp\left(\frac{t}{20}\right) + \frac{1}{2}exp\left(\frac{t}{10}\right)\right) \tag{30}$$

$$w(S,0) = S + 200\sqrt{S} + 100 \tag{31}$$

Thus, following the same procedure as in case I, by applying the PDTM with the parameters in case II through (30)-(31) gives the following:

$$W(S,0) = S + 200\sqrt{S} + 100 \tag{32}$$

$$W(S,1) = \frac{1}{2500}(6S^3 + 1200S^{\frac{5}{2}} + 600S^2 - 75S - 2500S^{\frac{1}{2}} + 5000) \tag{33}$$

$$\begin{aligned} W(S,2) =& \frac{9S^6}{781250} + \frac{36S^{\frac{11}{2}}}{15625} + \frac{18S^5}{15625} - \frac{9S^4}{62500} - \frac{3S^{\frac{7}{2}}}{625} + \frac{501S^3}{62500} \\ & - \frac{3411S^{\frac{5}{2}}}{15625} - \frac{222S^2}{625} - \frac{48S^{\frac{3}{2}}}{625} + \frac{9S}{5000} + \frac{\sqrt{S}}{100} - \frac{1}{25} \end{aligned} \tag{34}$$

$$\vdots$$

Whence,

$$
\begin{aligned}
w(S,t) =& \sum_{h=0}^{\infty} W(S,h)t^h \\
=& W(S,0) + W(S,1)t + W(S,2)t^2 + W(S,3)t^3 + \cdots \\
=& (S + 200\sqrt{S} + 100) \\
&+ \left(\frac{1}{2500} \left(6S^3 + 1200S^{\frac{5}{2}} + 600S^2 - 75S - 2500S^{\frac{1}{2}} + 5000 \right) \right) t \\
&+ \left(\frac{9S^6}{781250} + \frac{36S^{\frac{11}{2}}}{15625} + \frac{18S^5}{15625} - \frac{9S^4}{62500} - \frac{3S^{\frac{7}{2}}}{625} + \frac{501S^3}{62500} \right. \\
&\left. - \frac{3411S^{\frac{5}{2}}}{15625} - \frac{222S^2}{625} - \frac{48S^{\frac{3}{2}}}{625} + \frac{9S}{5000} + \frac{\sqrt{S}}{100} - \frac{1}{25} \right) t^2 + \cdots
\end{aligned}
\tag{35}
$$

In tables 1- 3, we present in comparison, the exact and the approximate solutions for time $t = 0, 0.5$ and 1 respectively. In addition, figure 3 and figure 4 below are the graphics for approximate and exact solutions for problem case II respectively, for $t \in [1,2]$ and $S \in [0.1, 5]$.

Table 1: t=0

S	w_{exact}	w_{approx}	$\left\lvert \frac{w_{exact} - w_{approx}}{w_{exact}} \right\rvert$
0.0	100.000	100.000	0.00000
0.5	241.921	241.921	1.2E-16
1.0	301.000	301.000	0.00000
1.5	346.449	346.449	0.00000
2.0	384.843	384.843	0.00000
2.5	418.728	418.728	1.4E-16
3.0	449.410	449.410	0.00000
3.5	477.666	477.666	1.2E-16
4.0	504.000	504.000	0.00000
4.5	528.764	528.764	0.00000
5.0	552.214	552.214	0.00000

Table 2: t=0.5

S	w_{exact}	w_{approx}	$\left\lvert \frac{w_{exact} - w_{approx}}{w_{exact}} \right\rvert$
0.0	105.127	101.990	0.0298
0.5	250.629	243.300	0.0292
1.0	311.190	302.528	0.0278
1.5	357.777	348.674	0.0254
2.0	397.130	388.385	0.0220
2.5	431.860	424.306	0.0175
3.0	463.307	457.863	0.0117
3.5	492.265	489.986	0.0046
4.0	519.253	521.378	0.0041
4.5	544.631	552.641	0.0147
5.0	568.662	584.355	0.0276

Table 3: t=1

S	w_{exact}	w_{approx}	$\left\lvert \frac{w_{exact} - w_{approx}}{w_{exact}} \right\rvert$
0.0	110.517	103.960	0.05933
0.5	259.689	244.590	0.05815
1.0	321.771	303.729	0.05607
1.5	369.525	350.145	0.05244
2.0	409.861	390.570	0.04707
2.5	445.458	427.789	0.03966
3.0	477.688	463.451	0.02980
3.5	507.367	498.813	0.01686
4.0	535.026	535.043	3.3E-05
4.5	561.034	573.390	0.02202
5.0	585.660	615.287	0.05059

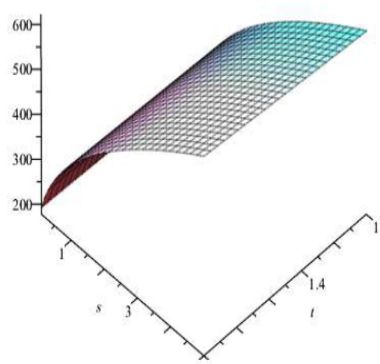

Figure 3: Exact solution for case II

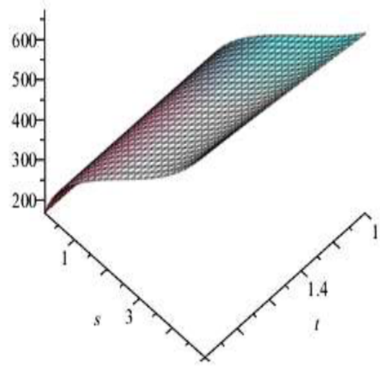

Figure 4: Approximate solution for case II

5. Concluding Remarks

In this paper, we considered a nonlinear transaction-cost model for stock prices in an illiquid market. This nonlinear model was arrived at when the constant volatility assumption of the classical linear Black-Scholes option pricing model was relaxed through the inclusion of transaction cost. We obtained approximate solutions to this nonlinear model using the projected differential transform method PDTM as a semi-analytical method. The results are very interesting, agree with the associated exact solutions obtained by Esekon (2013) and that of Gonzalez-Gaxiola et al. (2015) using the Adomian decomposition method; even though our approximate solutions include only terms up to time power two. All numerical computations and graphics done in this work were by Maple 18.

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